A ship is heading due west at a steady speed of 15 km/h. A current of 3 km/h is running due south.

a.) Find the resulting speed and direction of the ship.

\[ \sqrt{15^2 + 3^2} = 15.3 \text{ km/h} \]
\[ \theta = \tan^{-1}\left(\frac{3}{15}\right) = 11.3^\circ \]
\[ \text{Direction: } 11.3^\circ \text{ S of W} \]

b.) How many degrees north should the ship travel if it wants to head due west?

\[ \theta = \sin^{-1}\left(\frac{3}{15}\right) = 11.5^\circ \]
\[ \text{Direction: } 11.5^\circ \text{ N of W} \]

2. Two tractors pull on a large boulder in an effort to shift it out of the way of a new fence line. One tractor pulls with a force of 3000 N west and the other tractor pulls with a force of 2500 N in a southerly direction because of the terrain.

a.) Determine the resultant force acting on the boulder.

\[ 3000^2 + 2500^2 = x^2 \]
\[ x = 3905.1 \text{ N} \]

b.) Determine the resulting direction of the boulder.

\[ \theta = \tan^{-1}\left(\frac{2500}{3000}\right) = 39.8^\circ \]
\[ \text{Direction: } 39.8^\circ \text{ S of W} \]

3. A boat heads directly across a river at 8 mi/h. The river flows downstream at 10 mi/h. Find the resulting speed and direction of the boat.

\[ 12^2 = 8^2 + 10^2 \]
\[ k = 12.8 \text{ mph} \]
\[ \theta = \tan^{-1}\left(\frac{10}{8}\right) = 51.3^\circ \]
\[ 12.8 \text{ mph} (51.3^\circ \text{ S of E}) \]

4. A woman walks 6 miles due east from her home and then walks 8 mi due south.

a.) How far is she from her home?

\[ 6^2 + 8^2 = x^2 \]
\[ x = 10 \text{ miles} \]

b.) In what direction should she walk to return home?

\[ \theta = \tan^{-1}\left(\frac{8}{6}\right) = 56.9^\circ \]
\[ 36.9^\circ \text{ W of N} \]

5. A bird heads directly south at 5 mi/h encounters a wind blowing due east at 3 mi/h. How would the direction of the bird be affected?

\[ \theta = \tan^{-1}\left(\frac{3}{5}\right) = 31^\circ \]
\[ 31^\circ \text{ E of S or } 301^\circ \]

If you went west across the river then an answer of 12.8 mph (51.3\text{ S of W}) or 12.8 mph (231.3\text{ S}) would be correct.
Find the component form of the vector.

6. \( u + v \)
   \[ \begin{bmatrix} -1, 3 \end{bmatrix} + \begin{bmatrix} 2, 4 \end{bmatrix} = \begin{bmatrix} 1, 7 \end{bmatrix} \]

9. \( 3v \)
   \[ \begin{bmatrix} 1, 4 \end{bmatrix} \]

12. \(-2u - 3v\)
   \[ \begin{bmatrix} 2, -6 \end{bmatrix} \]

7. \( u - v \)
   \[ \begin{bmatrix} -1, 3 \end{bmatrix} - \begin{bmatrix} 2, 4 \end{bmatrix} = \begin{bmatrix} -3, -1 \end{bmatrix} \]

10. \(2u + 3w\)
    \[ \begin{bmatrix} -2, 6 \end{bmatrix} \]

13. \(-3v + 2w\)
    \[ \begin{bmatrix} 4, 10 \end{bmatrix} \]

15. Captain Bob is flying a plane and leaves Dayton traveling at 120 mph 25° North of West. There is a headwind of 10 mph at 35° South of West. What is the plane's resulting speed and direction?

   \[ \vec{v} = 120 \cos 25° \text{ mph} \]
   \[ \vec{w} = 10 \cos 35° \text{ mph} \]
   \[ \vec{v} + \vec{w} = \begin{bmatrix} -114.9, 44.978 \end{bmatrix} \]
   \[ b^2 = 114.9^2 + 44.978^2 \]
   \[ b = 125.3 \text{ mph} \]
   \[ \Theta = \tan^{-1} \left( \frac{44.978}{114.9} \right) = 21° \]
   \[ \text{Direction} = 180° - \Theta \]
   
16. If a boat wants to head 40° North of West at 20 mph, but there is a current pushing it South at 3 mph. What direction and speed should the boat head out at?

   \[ \vec{b} = \begin{bmatrix} h, v \end{bmatrix} \]
   \[ \vec{c} = \begin{bmatrix} 3 \cos 10°, 0, -3 \end{bmatrix} \]
   \[ \text{Resultant} = \begin{bmatrix} 20 \cos 140°, 20 \sin 140° \end{bmatrix} \]
   \[ h = 20 \cos 140° \]
   \[ v = 20 \sin 140° \]
   \[ \text{Resultant} = \begin{bmatrix} -15.321, 15.856 \end{bmatrix} \]
   \[ b = 22.0 \text{ mph} \]
   \[ \Theta = \tan^{-1} \left( \frac{15.856}{15.321} \right) = 45.98° \]
   \[ \text{Direction} = 180° - \Theta \]

17. A river naturally flows at a rate of 5 mph South. If a boat leaves dock and travel 42° South of East at 30 mph. What is the resulting speed and direction?

   \[ \vec{v} = \begin{bmatrix} 4.77 \text{ mph} \end{bmatrix} \]
   \[ \vec{b} = \begin{bmatrix} 30 \cos 42°, 30 \sin 42° \end{bmatrix} \]
   \[ \begin{bmatrix} 22.941, -25.074 \end{bmatrix} \]
   \[ b^2 = 22.941^2 + 25.074^2 \]
   \[ b = 33.6 \text{ mph} \]
   \[ \Theta = \tan^{-1} \left( \frac{25.074}{22.941} \right) = 48.4° \]
   \[ \text{Direction} = 360° - \Theta \]
   
18. If a boat wants to head 32° North of East at 40 mph, but there is a current pushing it South at 8 mph. What direction and speed should the board head out at?

   \[ \vec{b} = \begin{bmatrix} 40 \cos 32°, 40 \sin 32° \end{bmatrix} \]
   \[ \begin{bmatrix} 33.922, 29.197 \end{bmatrix} \]
   \[ b^2 = 33.922^2 + 29.197^2 \]
   \[ b = 44.8 \text{ mph} \]
   \[ \Theta = \tan^{-1} \left( \frac{29.197}{33.922} \right) = 40.7° \]
   \[ \text{Direction} = \theta \]
or # 1-6 use vectors $\vec{p} = 1.3\text{cm}(50^\circ)$ and $\vec{q} = 2.9\text{cm}(10^\circ)$

Find the components of the vector.
1. $\vec{p} = 1.3 \cos 50^\circ$, $1.3 \sin 50^\circ$
   \[ <0.836, 0.996> \]
2. $\vec{q} = 2.9 \cos 10^\circ$, $2.9 \sin 10^\circ$
   \[ <2.856, 0.504> \]

Find the resultant using the geometric parallelogram method.
3. $\vec{p} + \vec{q}$
   \[ k^2 = 1.3^2 + 2.9^2 - 2(1.3)(2.9)\cos 140^\circ \]
   \[ k = 3.984 \]
   \[ \sin \theta = \sin 140^\circ \frac{1.3}{3.984} \]
   \[ \theta = \sin^{-1} \left( \frac{1.3 \sin 140^\circ}{3.984} \right) \]
   \[ \theta = 12.11^\circ \]
   \[ \text{direction: } 10 + \theta \]
   \[ 3.98\text{ cm (22.11°)} \]

4. $2\vec{p} + \vec{q}$
   \[ k^2 = 2.9^2 + 2.6^2 - 2(2.9)(2.6)\cos 140^\circ \]
   \[ k = 5.169 \]
   \[ \sin \theta = \sin 140^\circ \frac{2.6}{5.169} \]
   \[ \theta = \sin^{-1} \left( \frac{2.6 \sin 140^\circ}{5.169} \right) \]
   \[ \theta = 18.863^\circ \]
   \[ \text{direction: } 10 + \theta \]
   \[ 5.17\text{ cm (28.86°)} \]

Find the resultant using the components method.
5. $3\vec{p} - \vec{q} = 3\vec{p} + -\vec{q}$
   \[ 3\vec{p} = <3.9 \cos 50^\circ, 3.9 \sin 50^\circ> = <2.508, 2.988> \]
   \[ -\vec{q} = <2.9 \cos 190^\circ, 2.9 \sin 190^\circ> = <-2.856, -0.504> \]
   \[ 3\vec{p} - \vec{q} = <-0.348, 2.484> \]
   \[ k^2 = (-0.348)^2 + (2.484)^2 \]
   \[ k = 2.508 \]
   \[ \theta = \tan^{-1} \left( \frac{2.484}{-0.348} \right) = 82.0^\circ \]
   \[ \text{direction: } 180 - \theta \]
   \[ 2.51\text{ cm (98.0°)} \]

6. $4\vec{p} - \vec{q}$
   \[ 4\vec{p} = <5.2 \cos 50^\circ, 5.2 \sin 50^\circ> = <3.344, 3.984> \]
   \[ -\vec{q} = <2.9 \cos 190^\circ, 2.9 \sin 190^\circ> = <-2.856, -0.504> \]
   \[ 4\vec{p} - \vec{q} = <3.488, 3.488> \]
   \[ k^2 = 0.488^2 + 3.488^2 \]
   \[ k = 3.514 \]
   \[ \theta = \tan^{-1} \left( \frac{3.488}{0.488} \right) = 22.0^\circ \]
   \[ \text{direction: } \theta \]
   \[ 3.51\text{ cm (82°)} \]
7. A small airplane lands at a point 246 miles east and 76 miles north of the point from which it took off. Describe the magnitude and the direction of its flight vector.

\[ r = \sqrt{246^2 + 76^2} = 257.5 \text{ Mi} \]
\[ \theta = \tan^{-1} \left( \frac{76}{246} \right) = 17.2^\circ \]

257.5 Mi (17.2°)

8. The speed of a powerboat in still water is 35 mph. It is traveling on a river that flows directly south at 8 mph.

a. If the boat's initial bearing is west, what are the resulting speed and direction of the boat?

\[ r = \sqrt{35^2 + 8^2} = 35.9 \text{ mph} \]
\[ \theta = \tan^{-1} \left( \frac{8}{35} \right) = 13.2^\circ \]

13.2° N of W

b. At what angle should the boat head upriver in order to travel directly west?

\[ \theta = \sin^{-1} \left( \frac{8}{35} \right) = 13.2^\circ \]

9. A twin-engine airplane has a speed of 300 mph in still air. Suppose this airplane heads directly south and encounters a 50 mph wind blowing due east. Find the resulting speed and direction of the plane.

\[ r = \sqrt{300^2 + 50^2} = 304.1 \text{ mph} \]
\[ \theta = \tan^{-1} \left( \frac{50}{300} \right) = 9.5^\circ \]

304.1 mph (279.5°)

10. A boat left dock A, traveled north for 10 miles, then 45° east of north for 20 miles and docked at B. Find the magnitude and direction for the resultant vector.

\[ r = \sqrt{10^2 + 20^2} = 22.4 \text{ Mi} \]
\[ \theta = \tan^{-1} \left( \frac{20}{10} \right) = 63.4^\circ \]

22.4 Mi (63.4° N of E)

11. An airplane is flying on a bearing of 335° at 530 mph. Find the component form of the velocity of the airplane.

\[ \vec{v} = 530 \cos 335^\circ, 530 \sin 335^\circ \]
\[ \vec{v} = \langle 450.8, -224.0 \rangle \]

12. A basketball is shot at a 70° angle with the horizontal direction with an initial speed of 10 m/sec. Find the component form of the initial velocity.

\[ \vec{v}_0 = 10 \cos 70^\circ, 10 \sin 70^\circ \]
\[ \vec{v}_0 = \langle 4.0, 9.4 \rangle \]

13. Find the magnitude and direction of the resultant. A force of 50 lb acts on an object at an angle of 45°. A second force of 75 lb acts on the object at an angle of 30°.

\[ \vec{f}_1 = 50 \cos 45^\circ, 50 \sin 45^\circ \]
\[ \vec{f}_2 = 75 \cos 30^\circ, 75 \sin 30^\circ \]

14. A ship heads due south with the current flowing northwest. Two hours later the ship is 20 miles in the direction 30° west of south from the original starting point. Find the speed with no current of the ship and the rate of the current.

\[ \text{Speed} = \frac{20}{2} = 10 \text{ mph} \]

\[ \frac{\sin 105^\circ}{10} = \frac{\sin 45^\circ}{s} \]
\[ s = 13.6 \text{ mph} \]

\[ \frac{\sin 30^\circ}{c} = \frac{\sin 45^\circ}{10} \]
\[ c = 7.07 \text{ mph} \]