Expand each logarithm

1. \( \log_2(9^5) = 5 \log_2 9 \)

2. \( \log_2(21) = \)

3. \( \log_5 \left( \frac{19}{2} \right) = \log_5 19 - \log_5 2 \)

4. \( \log_2(6a) = \log_2 6 + \log_2 a \)

5. \( \log_3(xy) = \log_3 x + \log_3 y \)

6. \( \log_5 \left( \frac{a}{3} \right) = \log_5 a - \log_5 3 \)

7. \( \log_3(5y) = \log_3 5 + \log_3 y \)

8. \( \log_3(a^{10}) = 10 \log_3 a \)

9. \( \log_5 \left( \frac{12a}{2} \right) = \log_5 12 + \log_5 a - \log_5 2 \)

10. \( \log_2 \left( \frac{a}{b} \right)^5 = \frac{5}{2} \log_2 a - \log_2 b \)

11. \( \log_5 \sqrt{x^5 y} = \frac{5}{2} (\log_5 x + \log_5 y) \)

12. \( \log_5 \left( \frac{xy}{z} \right)^8 = 8 \left( \log_5 x + \log_5 y - \log_5 z \right) \)

13. \( \log_2 \left( \frac{1-x}{y} \right)^3 = 3 (\log_2 (1-x) - \log_2 y) \)

14. \( \log_3 \sqrt[3]{9x^3} = \frac{1}{3} \log_3 9 + 3 \log_3 x \)

15. \( \log_3 \sqrt[3]{2x^5} = \frac{1}{3} (\log_3 2 + 5 \log_3 x) \)

16. \( \log_2 \left( \frac{9x^{10}}{y^2} \right) = \log_2 9 + 10 \log_2 x - 2 \log_2 y \)

17. \( \log_2 \left( \frac{4a}{5} \right) = \log_2 4 + \log_2 a - \log_2 5 \)

18. \( \log_2 \left( x^2 + 3 \log_2 a \right) \)

Rewrite each logarithm as a single logarithm

19. \( 2 \log_3 10 - \log_3 4 = \log_3 \left( \frac{10^2}{4} \right) = \log_3 \left( \frac{25}{4} \right) \)

20. \( \frac{2}{3} \log_2 x + \log_2 y = \log_2 \left( x^{2/3} y \right) \) or \( \log_2 \left( 3 \sqrt[3]{x^2} y \right) \)

21. \( \frac{1}{2} \log_5 x + \log_5 y = \log_5 \left( \sqrt[5]{x} y \right) \)

22. \( 3 \log_3 x + 4 \log_3 y = \log_3 \left( x^3 y^4 \right) \)

23. \( 6 \log_3 x + 2 \log_3 11 = \log_3 \left( 11 x^6 \right) \)

24. \( 4 \log_5 x - (\log_5 y + \log_5 2) = \log_5 \left( \frac{x^4}{y 2} \right) \)

25. \( \frac{1}{2} \log_3 144 - \log_3 4 = \log_3 \left( \frac{144^{1/2}}{4} \right) = \log_3 \left( \frac{12}{4} \right) = \log_3 3 \)

26. \( \log_3 a + \log_3 b - 2 \log_3 c = \log_3 \left( \frac{ab}{c^2} \right) \)
Solve each logarithmic equation.

27. \(\log 5x = \log(2x + 9)\)
   \[
   \frac{5x}{x} = 2x + 9
   \]
   \[
   3x = 9
   \]
   \[
   x = 3
   \]

28. \(\log(10 - 4x) = \log(10 - 3x)\)
   \[
   10 - 4x = 10 - 3x
   \]
   \[
   -x = 0
   \]
   \[
   x = 0
   \]

29. \(-6\log_3(x - 3) = -24\)
   \[
   \frac{-6}{x - 3} = -24
   \]
   \[
   3^4 = x - 3
   \]
   \[
   x = 84
   \]

30. \(-10 + \log_3(x + 3) = -8\)
   \[
   \log_3(x + 3) = 2
   \]
   \[
   3^2 = x + 3
   \]
   \[
   9 = x + 3
   \]
   \[
   x = 6
   \]

31. \(-2\log_5(7x) = 2\)
   \[
   \log_5(7x) = -1
   \]
   \[
   5^{-1} = 7x
   \]
   \[
   \frac{5}{7} = x
   \]
   \[
   x = \frac{5}{7}
   \]

32. \(-6\log_3(x - 3) = -24\)
   \[
   \log_3(x - 3) = 4
   \]
   \[
   3^4 = x - 3
   \]
   \[
   x = 84
   \]

33. \(\log_{12}(x^2 + 35) = \log_{12}(-12x - 1)\)
   \[
   x^2 + 35 = -12x - 1
   \]
   \[
   x^2 + 12x + 36 = 0
   \]
   \[
   (x + 6)^2 = 0
   \]
   \[
   x = -6
   \]

34. \(\ln(x^2 + 12) = \ln(-9x - 2)\)
   \[
   x^2 + 12 = -9x - 2
   \]
   \[
   x^2 + 9x + 14 = 0
   \]
   \[
   (x + 7)(x + 2) = 0
   \]
   \[
   x = -7, -2
   \]

35. \(\ln 2 - \ln(3x + 2) = 1\)
   \[
   \ln(\frac{2}{3x + 2}) = 1
   \]
   \[
   e^1 = \frac{2}{3x + 2}
   \]
   \[
   e \cdot (3x + 2) = 2
   \]
   \[
   3x + 2 = \frac{2}{e}
   \]
   \[
   x = \frac{2}{e} - 2
   \]

36. \(\ln(x - 3) - \ln(x - 5) = \ln 5\)
   \[
   \ln \left(\frac{x - 3}{x - 5}\right) = \ln 5
   \]
   \[
   \frac{x - 3}{x - 5} = 5
   \]
   \[
   x - 3 = 5(x - 5)
   \]
   \[
   x - 3 = 5x - 25
   \]
   \[
   -4x = -22
   \]
   \[
   x = \frac{22}{4} = \frac{11}{2}
   \]

37. \(\log_8 2 + \log_8 4x^2 = 1\)
   \[
   \log_8 (8x^2) = 1
   \]
   \[
   8^1 = 8x^2
   \]
   \[
   1 = x^2
   \]
   \[
   x = \pm 1
   \]

38. \(\ln(-3x - 1) - \ln 7 = 2\)
   \[
   \ln \left(\frac{-3x - 1}{7}\right) = 2
   \]
   \[
   e^2 = -3x - 1
   \]
   \[
   7e^2 = 3x + 1
   \]
   \[
   \ln \left(\frac{3x + 1}{7e^2}\right) = 1
   \]
   \[
   x = -1.75
   \]

39. \(\frac{5 \cdot 2^x}{5} = 60\)
   \[
   2^x = 12
   \]

40. \(e^{4x} + 5 = 30\)
   \[
   e^{4x} = 25
   \]
   \[
   \ln e^{4x} = \ln 25
   \]
   \[
   4x \ln e = \ln 25
   \]
   \[
   4x = \ln 25
   \]
   \[
   x = \frac{\ln 25}{4}
   \]
   \[
   x = 6.38
   \]

41. \(-9.7e^{7n-9} = -94\)
   \[
   e^{7n-9} = \frac{94}{9.7}
   \]
   \[
   \ln e^{7n-9} = \ln \left(\frac{94}{9.7}\right)
   \]
   \[
   (7n-9) \ln e = \ln \left(\frac{94}{9.7}\right)
   \]
   \[
   7n - 9 = \ln \left(\frac{94}{9.7}\right)
   \]
   \[
   n = \ln \left(\frac{94}{9.7}\right) + 9
   \]
   \[
   \ln = 1.11
   \]
Find the total value of the investment after the time given.

42. Sean invests $5,000 into an account that pays 4.625% interest compounded quarterly. What is the amount of money he has after 5 years?

\[
A = P \left(1 + \frac{r}{n}\right)^{nt}
\]
\[
A = 5000 \left(1 + \frac{0.04625}{4}\right)^{4 \cdot 5}
\]
\[
A = $6292.52
\]

43. Alex buys a ring for his girlfriend for $6000. At the rate of gold and diamonds are increasing, the ring increases its value at a rate of 3.1825% per year. If he plans on proposing in 2 years, how much will the ring be worth then?

\[
y = 6000 \left(1 + 0.031825\right)^x
\]
\[
y = 6000 \left(1 + 0.031825\right)^2
\]
\[
y = $6387.98
\]

44. Tyler invests $2500 into an account that pays 2% interest compounded continuously for his newborn son. If his son gets his money when he turns 18, how much will he have for college?

\[
A = Pe^{rt}
\]
\[
x = A = 2500e^{0.02 \cdot 18}
\]
\[
x = A = $3583.32
\]

45. Brian wants to have $10,000 for his son for college. He found an account online that pays 5% interest compounded continuously. How much money does he need to invest if his son will be 19 when he goes to college?

\[
10000 = Pe^{0.05 \cdot 19}
\]
\[
\frac{10000}{e^{0.05 \cdot 19}} = $3867.41
\]

46. A tractor cost $50,000 four years ago. Now it is worth $41,000. Find the annual rate of depreciation.

\[
y = a(1 + r)^x
\]
\[
41000 = 50000(1 - r)^4
\]
\[
0.82 = (1 - r)^4
\]
\[
0.18 = 1 - r
\]
\[
r = 0.48
\]
\[
r = 4.8\%
\]

47. Levi invested $50 on his 5th birthday and forgot about it for many years until there was $525 in the account. If the account paid 12% interest, compounded quarterly. How old is Levi now?

\[
A = P \left(1 + \frac{r}{n}\right)^{nt}
\]
\[
525 = 50 \left(1 + \frac{0.12}{4}\right)^{4t}
\]
\[
t = 19.88\text{ yrs}
\]
\[
\approx 25\text{ yrs old}
\]